Quantitative Finance

Joaquim Montezuma de Carvalho

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100 ANOS A PENSAR NO FUTURO

Review of Power Functions (or exponentials)

Bases and Exponents

A power function has the form:

$$b^n = \quad b \cdot b \cdot \ldots . \cdot b \,(\text{n times})$$

b **is known as the base, while** ***n*** **is called the power or exponent, and the power function means that** ***b*** **is multiplied by itself** ***n*** **times.**

Review of Power Functions

Combinations of Bases and Powers:

$$b^n \cdot e^n = (b \cdot e)^n$$

$$b^n \cdot b^m = b^{n+m} \qquad \frac{b^n}{b^m} = b^{n-m}$$

$$(b^m)^n = b^{m \cdot n}$$

Review of Power Functions

Particular Exponent Values:

$$b^0 = 1$$

$$b^{-n} = \frac{1}{b^n}$$

$$b^{\frac{1}{n}} = \sqrt[n]{b}$$

$$b^{\frac{m}{n}} = \sqrt[n]{b^m}$$

Logarithms

The function that is the **inverse** of the power function, which is called a **logarithm:**

$$\log_n a = b \text{ if } n^b = a$$

The base-10 logarithm is called the **common logarithm**, and the subscript is typically dropped:

$\log_{10} a = \log a$

The base-*e* logarithm, the inverse of *ex*, is called the **natural logarithm**, and it is typically abbreviated **ln**:

$\log_e a = \ln a$

Logarithms

$$\log_n(a \cdot b) = \log_n a + \log_n b$$

$$\log_n(\frac{a}{b}) = \log_n a - \log_n b$$

$$\log_n(a^m) = m \cdot \log_n a$$

$$\log_n 1 = 0$$

$$\log_n(\frac{1}{a}) = -\log_n a$$

$$\log_n(\sqrt[m]{a}) = \log_n(a^{\frac{1}{m}}) = \frac{1}{m}\log_n a$$

$$\log_n n = 1$$

Linear Interpolation

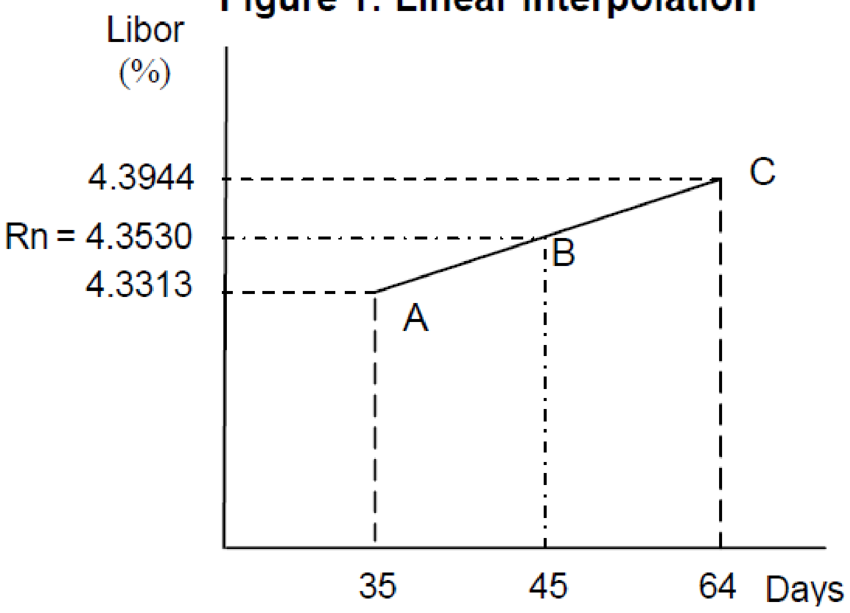
Figure 1 shows the relationship between the two rates and days to maturity. Linear interpolation assumes that the unknown rate (Rn) lies on the line (AC) between the two known rates. Because AC is linear, that is, a straight line, the slope of the line (AB) connecting R1 and Rn is the same as the slope of line AC. Using the "rise over run" formula for the slope of the line, we solve for Rn as follows:

$$Rn = R1 + \frac{R2 - R1}{t2 - t1} \times (tn - t1)$$

$$= 4.3313\% + \frac{4.3944\% - 4.3313\%}{64 - 35} \times (45 - 35)$$

$$= 4.3313\% + 0.00218\% \times (10) = 4.3530\%$$

Figure 1: Linear interpolation



The interpolated rate is 4.3530%, which lies between the two known rates.